

Mesa College – Math 104 (Trig) Challenge Exam SAMPLES

Directions: **NO CALCULATOR**. Write neatly and show your work and steps. Answers without appropriate work shown will receive little or NO credit. **Be sure to simplify all radicals and fractions.** Attach your neat and organized solution sheets behind this cover sheet. Make sure each solution is properly labeled.

#1 – 3. Use the given information to find the values of the remaining Trigonometric functions.

1. If $\sin \theta = -\frac{1}{2}$ and

$$\cos \theta = \frac{\sqrt{3}}{2}$$

2. If $\sin \theta = \frac{3}{5}$ and θ is in QII,

3. If $\cot \theta = -4$ and $270^\circ \leq \theta < 360^\circ$

#4-6. Find ALL values of x. Express your answers using radians.

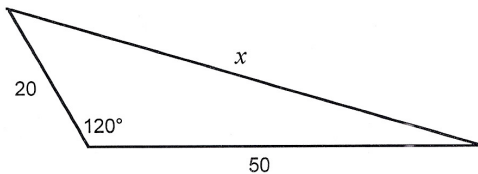
4. $3 \csc x - 6 = 0$.

5. $\tan^2 x = -\tan x$

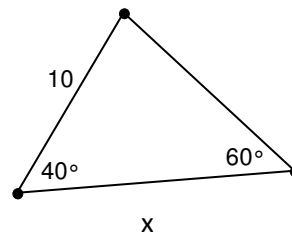
6. $2 \sin^2 x - 5 \sin x - 3 = 0$

7. Find the side length x in each of the triangles shown.

(7a)



(7b)



8. If $\cos x = 0.8$ and $\sin x = -0.6$, find :

(8a) $\sin(2x)$

(8b) $\cos(2x)$

(8c) in which quadrant is angle $(2x)$?

9. Given that $\cos \alpha = \frac{2}{3}$ and $\sin \beta = -\frac{1}{4}$, where $-\frac{\pi}{2} \leq \beta \leq \frac{\pi}{2}$ and $0 \leq \alpha \leq \pi$ find :

(9a) $\sin(\alpha + \beta)$

(9b) $\cos(\alpha - \beta)$

(9c) $\cos\left(\frac{\alpha}{2}\right)$

(9d) in which quadrant is angle $(\alpha + \beta)$?

(9e) in which quadrant is angle $(\alpha - \beta)$?

(9f) in which quadrant is angle $\left(\frac{\alpha}{2}\right)$?

10. Find the principal value of each:

10a) $\text{Arc cos}\left(\frac{-\sqrt{3}}{2}\right)$

10b) $\sin^{-1}\left(-\frac{1}{2}\right)$

10c) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

10d) $\text{arc cot}(\sqrt{3})$

10e) $\cos\left(\arcsin\left(\frac{\sqrt{3}}{2}\right)\right)$

10f) $\cos(\arctan(-1))$

10g) $\cot^{-1}\left(\sin\frac{3\pi}{2}\right)$

11. Find the **exact values** of each:

11a) $\sin\left(\frac{\pi}{4}\right)$

11b) $\cos(30^\circ)$

11c) $\csc\left(\frac{-3\pi}{4}\right)$

11d) $\sec(-420^\circ)$

11e) $\cot\left(\frac{13\pi}{6}\right)$

11f) $\frac{\sin\left(\frac{\pi}{6}\right)}{1+\cos\left(\frac{\pi}{6}\right)}$

11g) $\cos(45^\circ)\cos(60^\circ) - \sin(45^\circ)\sin(60^\circ)$

12. Simplify each expression completely.

(12a) $\sin(75^\circ)\cos(15^\circ) - \cos(75^\circ)\sin(15^\circ)$

(12b) $\sin^2\left(\frac{3\pi}{8}\right) - \cos^2\left(\frac{3\pi}{8}\right)$

(12c) $\cos\left(\frac{5\pi}{8}\right)$

13. Find all solutions between $0 \leq \theta < 360^\circ$ for each:

(13a) $\cos^2 \theta - 2\sin \theta = -2$

(13b) $\csc\left(\frac{x}{2}\right) = \sqrt{2}$

(13c) $\sin(2x) = \cos(x)$

14. Sketch at least one period of the graph of each. Label the coordinates of one maximum point and the coordinates of one minimum point.

(14a) $f(x) = -2\cos\left(x + \frac{\pi}{2}\right) + 1$

(14b) $g(x) = 5\sin 2\left(x - \frac{\pi}{4}\right)$

(14c) $h(x) = 3 + 2\sec x$

15. Simplify the expression $\frac{\cot y - 1}{1 - \tan y}$, so that it matches one of the expressions below. SHOW STEPS.

(a) $\cos y$

(b) $\tan y$

(c) 0

(d) $\frac{\sec y}{\csc y}$

(e) $\cot y$

16. Simplify each expression.

(16a) $(\sec \beta - \tan \beta)(\sec \beta + \tan \beta)$

(16b) $\frac{\cos \theta}{1 + \sin \theta} + \frac{1 + \sin \theta}{\cos \theta}$

17. Complete the **trigonometric identity**: $\sec \theta - \cos \theta = w$, by selecting w from the list of expressions below.

(a) $\tan \theta \sin \theta$

(b) $\cot^2 \theta \cos \theta$

(c) $\frac{1 - \cos \theta \sin \theta}{\sin \theta}$

(d) $\frac{\sec \theta - 1}{\sec \theta}$

(e) -1

18. SIMPLIFY each to a single trig expression involving no fractions.

(18a) $1 - \frac{\tan^2 x}{1 + \tan^2 x}$

(18b) $\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)^2 - 1$

19. If $A = 3 + 2i$ and $B = 5 - i$, where $i = \sqrt{-1}$, find each:

(19a) A^2 (19b) AB (19c) $\frac{A}{B}$

20. Evaluate: $2i^2 - 5i^{75} + \frac{6}{i^{13}}$, where $i = \sqrt{-1}$.

21. Express each in standard form (i.e. $a \pm bi$)

(21a) $5\sqrt{2}\left(\cos\left(\frac{5\pi}{4}\right) - i\sin\left(\frac{5\pi}{4}\right)\right)$ (21b) $8cis30^\circ$

22. Convert each from Polar coordinates to Cartesian coordinates

(22a) $(2, 30^\circ)$ (22b) $\left(-6, \frac{4\pi}{3}\right)$

23. Convert each from Cartesian coordinates to Polar coordinates

(23a) $(3, \sqrt{3})$ (23b) $(-5, -5)$

24. If $4 + 3i \approx 5cis37^\circ$, find $(4 + 3i)^4$ in trigonometric form.

Answers to Math 104 (Trig) SAMPLES

1. $\sin \theta = -\frac{1}{2}$, $\cos \theta = \frac{\sqrt{3}}{2}$, $\tan \theta = -\frac{\sqrt{3}}{3}$, $\csc \theta = -2$, $\sec \theta = \frac{2\sqrt{3}}{3}$, $\cot \theta = -\sqrt{3}$

2. $\sin \theta = \frac{3}{5}$, $\cos \theta = -\frac{4}{5}$, $\tan \theta = -\frac{3}{4}$, $\csc \theta = \frac{5}{3}$, $\sec \theta = -\frac{5}{4}$, $\cot \theta = -\frac{4}{3}$

3. $\sin \theta = -\frac{\sqrt{17}}{17}$, $\cos \theta = \frac{4\sqrt{17}}{17}$, $\tan \theta = -\frac{1}{4}$, $\csc \theta = -\sqrt{17}$, $\sec \theta = \frac{\sqrt{17}}{4}$, $\cot \theta = -4$

4. $x = \frac{\pi}{6} + 2\pi k$, $\frac{5\pi}{6} + 2\pi k$

5. $x = \pi k, \frac{3\pi}{4} + \pi k$

6. $x = \frac{7\pi}{6} + 2\pi k, \frac{11\pi}{6} + 2\pi k$

7. a. $x = 10\sqrt{39}$ b. $\frac{20\sqrt{3} \sin 80^\circ}{3}$

8. a. -0.96 b. 0.28 c. QIV

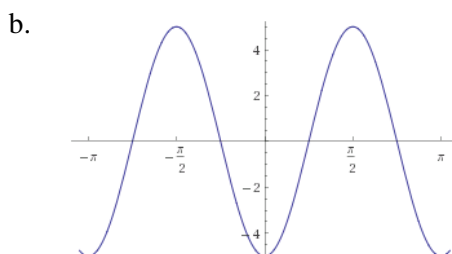
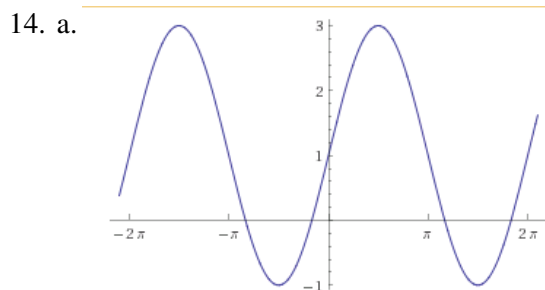
9. a. $\frac{5\sqrt{3}-2}{12}$ b. $\frac{2\sqrt{15}-\sqrt{5}}{12}$ c. $\frac{\sqrt{30}}{6}$ d. QI e. QI f. Q1

10. a. $\frac{5\pi}{6}$ b. $-\frac{\pi}{6}$ c. $\frac{\pi}{6}$ d. $\frac{\pi}{6}$ e. $\frac{1}{2}$ f. $\frac{\sqrt{2}}{2}$ g. $\frac{3\pi}{4}$

11. a. $\frac{\sqrt{2}}{2}$ b. $\frac{\sqrt{3}}{2}$ c. $-\sqrt{2}$ d. 2 e. $\sqrt{3}$ f. $2 - \sqrt{3}$ g. $\frac{\sqrt{2}-\sqrt{6}}{4}$

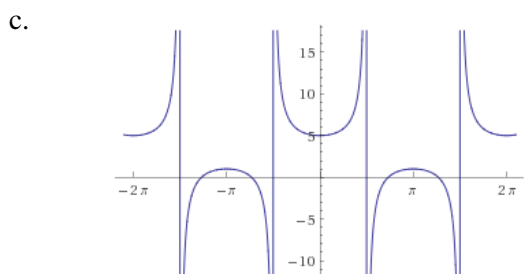
12. a. $\frac{\sqrt{3}}{2}$ b. $\frac{\sqrt{2}}{2}$ c. $\frac{\sqrt{2-\sqrt{2}}}{2}$

13. a. 90° b. $90^\circ, 270^\circ$ c. $30^\circ, 90^\circ, 150^\circ, 270^\circ$



max: $(\frac{3\pi}{2}, 3)$ min: $(\frac{\pi}{2}, -1)$

max: $(\frac{\pi}{2}, 5)$, min: $(\pi, -5)$



15. e

16. a. 1 b. $2 \sec \theta$

17. a

18. a. $\cos^2 x$ b. $\sin x$

19. a. $5 + 12i$ b. $17 + 7i$ c. $\frac{1}{2} + \frac{1}{2}i$

20. $-2 - i$

21. a. $-5 + 5i$ b. $4\sqrt{3} + 4i$

22. a. $(\sqrt{3}, 1)$ b. $(3\sqrt{3}, 3\sqrt{3})$

23. a. $(2\sqrt{3}, \frac{\pi}{6})$ b. $(\frac{5\sqrt{2}, 5\pi}{4})$

24. $625(\cos 148^\circ + i\sin 148^\circ)$